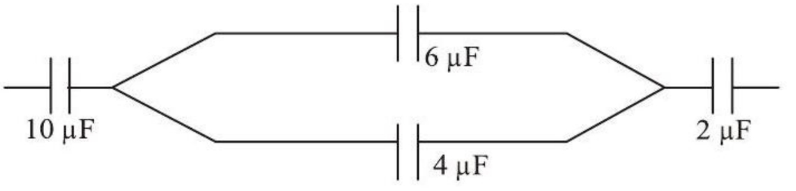
Electrostatic Potential and Capacitance

1. In the figure shown below, the charge on the left plate of the 10μ F capacitor is $-30\mu C$. The charge (in μC) on the right plate of the 6μ F capacitor is :



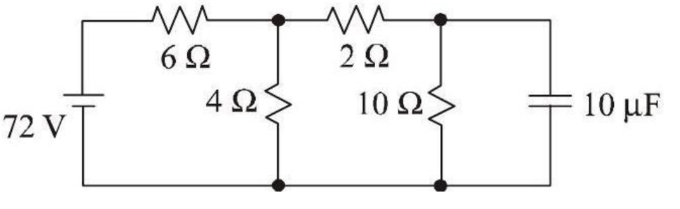
2. A parallel plate capacitor with plates of area $1\ m^2$ each, are at a separation of $0.1\ m$. If the electric field between the plates is $100\ N/C$, the magnitude of charge (in coulomb) on each plate is:
(Take $\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N-M^2}$)

3. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of $10^6\ V/m$. The plate area is $10^{-4}\ m^2$. What is the dielectric constant if the capacitance is 15 pF ? (given $\epsilon_0 = 8.86 \times 10^{-12} C^2\ m^2$)

4. A parallel plate capacitor has 1μ F capacitance. One of its two plates is given $+2\mu C$ charge and the other plate, $+4\mu C$ charge. The potential difference (in volt) developed across the capacitor is :

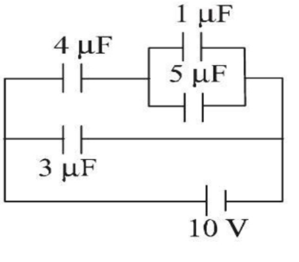
5. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{\imath}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ (in volt) is :

6. Determine the charge (in coulomb) on the capacitor in the following circuit:



7. A capacitor with capacitance 5μ F is charged to $5\mu C$. If the plates are pulled apart to reduce the capacitance to 2μ F, how much work (in joule) is done?

8. In the given circuit, the charge (in μC) on 4μ F capacitor will be :



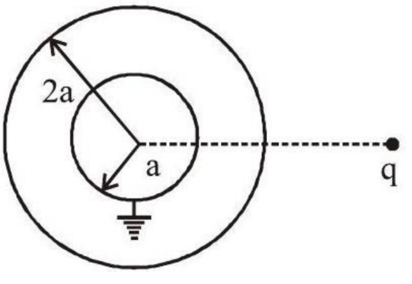
9. The electric potential is $V = (x^2 - 2x)$. What is the electric field strength at $x = 1$?

10. The 1000 small droplets of water each of radius r and charge Q, make a big drop of spherical shape. The potential of big drop is how many times the potential of one small droplet?

11. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential (in volt) at a distance of 2 cm from the centre of the sphere is

12. From a point charge, there is a fixed point A at some distance. At A, there is an electric field of $500 \mathrm{~V/m}$ and potential difference of 3000 V. Distance (in metre) between point charge and A will be

13. A solid conducting sphere of radius a is surrounded by a thin uncharged concentric conducting shell of radius 2 a. A point charge q is placed at a distance $4a$ from common centre of conducting sphere and shell. The inner sphere is then grounded. The charge on solid sphere is $\frac{q}{x}$. Find the value of x.

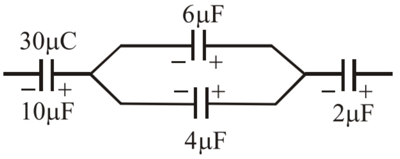


14. A 20μ F capacitor is connected to 45 V battery through a circuit whose resistance is 2000Ω. What is the final charge (in coulomb) on the capacitor?

15. Calculate the area (in m^2) of the plates of a one farad parallel plate capacitor if separation between plates is 1 mm and plates are in vacuum.

SOLUTIONS

1. **(18)**



As given in the figure, 6μF and 4μF are in parallel. Now using charge conservation

Charge on 6μF capacitor $= \frac{6}{6+4} \times 30$

$= 18\mu C$

Since charge is asked on right plate therefore is $+18\mu C$

2. **(8.85×10^{-10})** $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

$\therefore Q = \varepsilon_0 . E . A = 8.85 \times 10^{-12} \times 100 \times 1$

$= 8.85 \times 10^{-10} C$

3. **(8.5)** Capacitance of a capacitor with a dielectric of dielectric constant k is given by

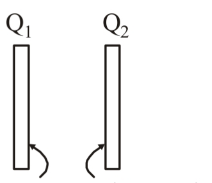
$$C = \frac{k \in_0 A}{d}$$

$$\because E = \frac{V}{d} \qquad \therefore C = \frac{k \in_0 AE}{V}$$

$$15 \times 10^{-12} = \frac{k \times 8.86 \times 10^{-12} \times 10^{-4} \times 10^{6}}{500}$$

$k = 8.5$

4. **(1)** $V = \frac{Q}{C}$



$\frac{Q_1 - Q_2}{2} - \frac{(Q_1 - Q_2)}{2}$

$= \left(\frac{Q_1 - Q_2}{2C}\right)$

$= \left(\frac{4-2}{2 \times 1}\right) = 1$ V

5. **(180)** Given, $\vec{E} = (Ax + B)\hat{i}$

or $E = 20x + 10$

Using $V = \int E dx$, we have

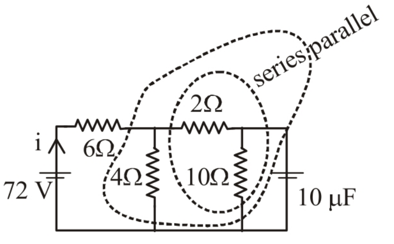
$$V_2 - V_1 = \int_{-5}^{1} (20x + 10) dx = -180 \text{ V}$$

or $V_1 - V_2 = 180$ V

6. **(200)** At steady state, there is no current in capacitor.

2Ω and 10Ω are in series. There equivalent resistance is 12Ω. This 12Ω is in parallel with 4Ω and there combined resistance is $12 \times 4/(12+4)$. This resistance is in series with 6Ω. Therefore, current drawn from battery

$$i = \frac{V}{R} = \left(\frac{72}{6 + \frac{12 \times 4}{12+4}} \right) = 8A$$



Current in 10Ω resistor

$$i' = \left(\frac{4}{4+12} \right) 8 = 2A$$

Pd across capacitor, $V = i'R = 2 \times 10 = 20V$

∴ Charge on the capacitor, $q = CV$

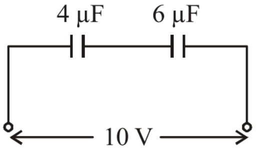
$= 10 \times 20 = 200\ \mu C$.

7. **(3.75×10^{-6})** $W = U_f - U_i = \frac{q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i} \right) \left(\because U = \frac{q^2}{2C} \right)$

$$= \frac{(5 \times 10^{-6})^2}{2} \left(\frac{1}{2} - \frac{1}{5} \right) \times 10^6$$

$= 3.75 \times 10^{-6}$ J

8. **(24)** $V_1 + V_2 = 10$



and $4V_1 = 6V_2$

On solving above equations, we get

$V_1 = 6$ V

Charge on 4 μf,

$q = CV_1 = 4 \times 6 = 24\ \mu C$

9. **(0)** $E = -\frac{dV}{dx} = -(2x-2)$

Now, E [at x = 1] = $-(2\times1-2) = 0$

10. **(100)** Volume of big drop = 1000 × volume of each small drop

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow R = 10r$

$\because \quad V = \frac{kq}{r}$ and $V' = \frac{kq}{R} \times 1000$

Total charge on one small droplet is q and on the big drop is 1000q.

$\Rightarrow \quad \frac{V'}{V} = \frac{1000r}{R} = \frac{1000}{10} = 100$

$\therefore \quad V' = 100V$

11. **(10)** Potential at any point inside the sphere = potential at the surface of the sphere = 10V.

12. **(6)** E = 500 V/m $\quad \Delta V = 3000$ V.

We know that electric field $|E| = 500 = \frac{\Delta V}{\Delta d}$

or $\Delta d = \frac{3000}{500} = 6\,m$

13. **(4)** The inner sphere is grounded, hence its potential is zero. The net charge on isolated outer sphere is zero. Let the charge on inner sphere be q'.

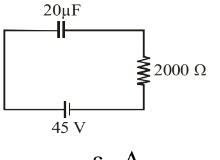
$\therefore$ Potential at centre of inner sphere is

$= \frac{1}{4\pi\varepsilon_0}\frac{q'}{a} + 0 + \frac{1}{4\pi\varepsilon_0}\frac{q}{4a} = 0 \quad \therefore \quad q' = -\frac{q}{4}$

14. **(9×10^{-4})** Final charge on the capacitor,

Q = CV

$= 20 \times 10^{-6} \times 45 = 9 \times 10^{-4}$C.



15. **(1.13×10^8)** For a parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$

$\therefore \quad A = \frac{Cd}{\varepsilon_0} = \frac{1\times10^{-3}}{8.85\times10^{-12}} = 1.13 \times 10^8\ m^2$

This corresponds to area of square of side 10.6 km which shows that one farad is very large unit of capacitance.